Homework 25

## Circular motion. Centripetal acceleration.

We are discussing circular motion. We already know that any motion along a curved line is accelerated motion. I would say that the simplest example of a curved line is a circle. Studying of the circular motion is even more important and general than it seems, because any smooth curved line can be composed from "pieces" of circles with different diameters. So the motion of an object along such curve at any moment of time can be represented as circular motion.


Figure 1.
A convenient way to describe the position of an object moving along a circular path is to measure the angle (let us denote it $\boldsymbol{\alpha}$ ) swiped by the radius connecting the object and the center of the circular path.

## A good way to measure angle.

There is a convenient way to measure the angle. We can divide the length of the circular arc swiped by the radius of the circle to the radius. The obtained quotient does not depend on the radius of the circle. The unit of the angle measured this way is called radian. For example the angle of $360^{\circ}$, corresponding to one complete turn is: $\alpha=2 \pi \mathbf{R} / \mathbf{R}=\mathbf{2} \pi \mathrm{rad}$. So one radian is approximately equal to $57.3^{\circ}$.

## Since circular motion is accelerated motion it does not persist without a force.

1. An overhead view of a person swinging a rock on a rope. A force from the string is required to make the rock's velocity vector keep changing direction.
2. If the string breaks, the rock will follow Newton's first law and go straight instead of continuing around the circle.


Figure 2 (www.lightandmatter.com)

## The force is directed inward and called centripetal force.

Figure 2 showed the string pulling in straight along a radius of the circle, but many people believe that when they are doing this they must be "leading" the rock a little to keep it moving along. That is, they believe that the force required to produce uniform circular motion is not directly inward but at a slight angle to the radius of the circle. This intuition is incorrect, which you can easily verify for yourself now if you have some string handy. It is only while you are getting the object going that your force needs to be at an angle to the radius. During this initial period of speeding up, the motion is not uniform. Once you settle down into uniform circular motion, you only apply an inward force.


Fig. 3 (www.lightandmatter.com)

A series of three hammer taps makes the rolling ball trace a triangle, seven hammers a heptagon. If the number of hammers was large enough, the ball would essentially be experiencing a steady inward force, and it would go in a circle. In no case is any forward force necessary.

1. A plane is moving along a circular path (or making a turn). Draw the force diagram and show which force (or forces) produce centripetal acceleration.
2. You are staying at the equator. Calculate your linear speed with respect to the center of Earth.
3. Let us imagine that the Earth rotation axis is perpendicular to the Earth's orbit plane (the ecliptic). Let us also imagine that your house is at the equator. You measure the speed of your house with respect to the Sun at noon and at midnight. Will you get 2 different numbers? If yes, what is the difference?
