

WORK DONE BY IDEAL GAS.

MAY 12, 2024

THEORY RECAP

Molar mass. How do we know the amount of substance of some matter? It is really hard to resolve individual molecules, and there are too many of them (remember, the Avogadro number is $6 \cdot 10^{23}$) to count them one by one. But every molecule of a substance has the same mass, so $6.02 \cdot 10^{23}$ of these molecules have a particular mass which is characteristic of this substance. It is called molar mass and denoted by M . Mass is easy to measure and if we have m kilograms of substance with molar mass M , the amount of substance is

$$n = \frac{m}{M}.$$

Equation of state of ideal gas can now be written in terms of mass and molar mass:

$$pV = \frac{m}{M}RT$$

Molar mass from periodic table. How do we know the molar mass of some substance? As we have seen, molar mass is related to mass of molecules of the substance. Mass of a molecule is equal to sum of masses of atoms comprising this molecule. And masses of atoms can be found in the periodic table of elements, which contains a lot of useful information about all the atoms.

Periodic Table of the Elements

Atomic Number	Symbol	Name	Atomic Mass		
1	H	Hydrogen	1.008		
2	He	Helium	4.003		
3	Li	Lithium	6.941		
4	Be	Beryllium	9.012		
5	B	Boron	10.811		
6	C	Carbon	12.011		
7	N	Nitrogen	14.007		
8	O	Oxygen	15.999		
9	F	Fluorine	18.998		
10	Ne	Neon	20.180		
11	Na	Sodium	22.990		
12	Mg	Magnesium	24.305		
13	Al	Aluminum	26.982		
14	Si	Silicon	28.086		
15	P	Phosphorus	30.974		
16	S	Sulfur	32.065		
17	Cl	Chlorine	35.453		
18	Ar	Argon	39.948		
19	K	Potassium	39.098		
20	Ca	Calcium	40.078		
21	Sc	Scandium	44.956		
22	Ti	Titanium	47.88		
23	V	Vanadium	50.942		
24	Cr	Chromium	51.996		
25	Mn	Manganese	54.938		
26	Fe	Iron	55.833		
27	Co	Cobalt	58.933		
28	Ni	Nickel	58.693		
29	Cu	Copper	63.546		
30	Zn	Zinc	65.39		
31	Ga	Gallium	69.723		
32	Ge	Germanium	72.61		
33	As	Arsenic	74.922		
34	Se	Selenium	78.972		
35	Br	Bromine	79.904		
36	Kr	Krypton	83.80		
37	Rb	Rubidium	84.464		
38	Sr	Strontium	87.62		
39	Y	Yttrium	88.906		
40	Zr	Zirconium	91.224		
41	Nb	Niobium	92.906		
42	Mo	Molybdenum	95.95		
43	Tc	Technetium	98.907		
44	Ru	Ruthenium	101.07		
45	Rh	Rhodium	102.906		
46	Pd	Palladium	106.42		
47	Ag	Silver	107.868		
48	Cd	Cadmium	112.411		
49	In	Indium	114.818		
50	Sn	Tin	118.71		
51	Sb	Antimony	121.760		
52	Te	Tellurium	127.6		
53	I	Iodine	126.904		
54	Xe	Xenon	131.29		
55	Cs	Cesium	132.905		
56	Ba	Barium	137.327		
57-71	Lanthanide Series				
72	Hf	Hafnium	178.49		
73	Ta	Tantalum	180.948		
74	W	Tungsten	183.85		
75	Re	Rhenium	186.207		
76	Os	Osmium	190.23		
77	Ir	Iridium	192.22		
78	Pt	Platinum	195.08		
79	Au	Gold	196.967		
80	Hg	Mercury	200.59		
81	Tl	Thallium	204.383		
82	Pb	Lead	207.2		
83	Bi	Bismuth	208.980		
84	Po	Polonium	[209]		
85	At	Astatine	[209]		
86	Rn	Radon	[222]		
87	Fr	Francium	[223]		
88	Ra	Radium	[226]		
89-103	Actinide Series				
104	Rf	Rutherfordium	[261]		
105	Db	Dubnium	[262]		
106	Sg	Seaborgium	[266]		
107	Bh	Bohrium	[264]		
108	Hs	Hassium	[269]		
109	Mt	Meitnerium	[268]		
110	Ds	Darmstadtium	[269]		
111	Rg	Roentgenium	[272]		
112	Cn	Copernicium	[277]		
113	Uut	Ununtrium	unknown		
114	Fl	Flerovium	[289]		
115	Uup	Ununpentium	unknown		
116	Lv	Livermorium	[293]		
117	Uus	Ununseptium	unknown		
118	Uuo	Ununoctium	unknown		

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There is a simple algorithm of finding molar mass from periodic table. First of all, we need to locate atomic mass in the periodic table: it is the lowest number in each cell. For example, for the first element - hydrogen (H) we can see the atomic mass is 1.008 which

could be rounded to 1. This is exactly the molar mass of a hydrogen atom, measured in gram/mole. So, if we take 1 mole of hydrogen atoms, or $6 \cdot 10^{23}$ hydrogen atoms, their mass will be $M(\text{H}) \cdot 1 \text{ mole} = 1 \text{ g}$. If we take 1 mole of carbon atoms, their mass will be $M(\text{C}) \cdot 1 \text{ mole} = 12 \text{ g}$ (find carbon C in the table above and verify that its' atomic mass is about 12).

If instead of atoms we consider molecules, molar mass is sum of molar masses of atoms building the molecules. For example, a nitrogen molecule N_2 consists of two nitrogen atoms. Therefore, the molar mass of nitrogen molecules is twice the molar mass of nitrogen atoms:

$$M(\text{N}_2) = 2 \cdot M(\text{N}) = 2 \cdot 14 \text{ g/mole} = 28 \text{ g/mole}.$$

Let us do one more example. Consider a carbon dioxide molecule CO_2 which consists of a carbon atom and two oxygen atoms. From the periodic table we find that the molar mass of carbon atom C is 12 g/mole and that the molar mass of oxygen atom O is 16 g/mole. So we find molar mass of carbon dioxide:

$$M(\text{CO}_2) = M(\text{C}) + 2 \cdot M(\text{O}) = 12 + 2 \cdot 16 \text{ g/mole} = 44 \text{ g/mole}.$$

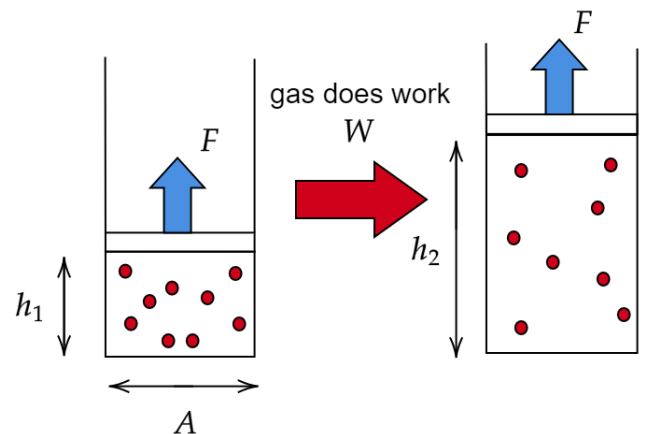
Work done by gas. Today we are considering a connection between mechanics and thermodynamics. We will learn how heat can be transferred to work using a gas.

First, a reminder: what is work? As you recall from mechanics, work is force times displacement. So in order to have work, we need to have force and displacement.

Consider our favorite setup: some amount of gas in a container under a piston. If cross section area of the container (and the piston) is A and gas pressure is p , the force with which gas acts on the piston follows from definition of pressure:

$$p = \frac{F}{A} \implies F = pA.$$

Let the gas be expanding, so the piston moves up by height $h = h_2 - h_1$ (see figure) above its initial position. Work done by the gas on the piston in this case is



$$W = Fh = pA(h_2 - h_1) = p(Ah_2 - Ah_1) = p(V_2 - V_1) = p\Delta V.$$

We used the fact that product of cross section area and height is volume of the gas (final volume is denoted V_2 and initial volume V_1) and denoted change of volume by ΔV . To sum up, we have derived the formula for work done by gas: work is done whenever volume is changed and for processes at constant pressure it is equal to pressure times change in volume:

$$W = p\Delta V.$$

Remember that work can be positive or negative: it is positive when force and displacement point in the same direction and negative when they point in the opposite directions. Let us consider two possible cases to show that our formula for work of the gas does get this right. When volume changes, it either increases or decreases. When volume increases, force on the piston and displacement of the piston both point up in our figure above. Therefore

work should be positive - and from our formula $W = p\Delta V > 0$ because $\Delta V > 0$ (and p is always positive). On the other hand when volume decreases, displacement is in the opposite direction (down on our figure) than the force (which still points up). Therefore work should be negative and from our formula $W = p\Delta V < 0$ because $\Delta V < 0$.

HOMWORK

1. There is a 1 liter bottle filled with water at 27°C. The water is liquid at this temperature because there is attracting force between the molecules. Imagine that we have suddenly “turned off” this attracting force. What is the pressure in the bottle now?
Hint: mass of 1 liter of water is 1 kg, molar mass of water is 18 grams/mole.
2. Find molar mass of molecular oxygen O_2 using periodic table. Using it, find the mass of oxygen in a 10 liter cylinder if it has temperature $T=13^\circ\text{C}$ and pressure $P = 9 \cdot 10^6$ Pa (note that it is 90x the normal atmospheric pressure!). For how long can the oxygen in this cylinder sustain a scuba diver, if an average person needs to inhale about 2 grams of oxygen per minute?
3. There is a cylinder with a piston. The mass of the piston is 100 kg, its area is 10 cm^2 . The cylinder contains 32 grams of oxygen at $T_1 = 273 \text{ K}$. The cylinder is heated up to $T_2 = 373 \text{ K}$. How does the piston position change? How does the potential energy of the piston change? What work is done by the gas? Neglect atmospheric pressure.
Hint: what is the gas pressure? Does it change?
- *4. How much hydrogen H_2 (in grams) is in a cylinder with a piston if it performs work of 400 J being heated from 250 K to 680 K? The gas pressure was maintained constant.