WORK DONE BY IDEAL GAS.

MAY 12, 2024

Theory recap

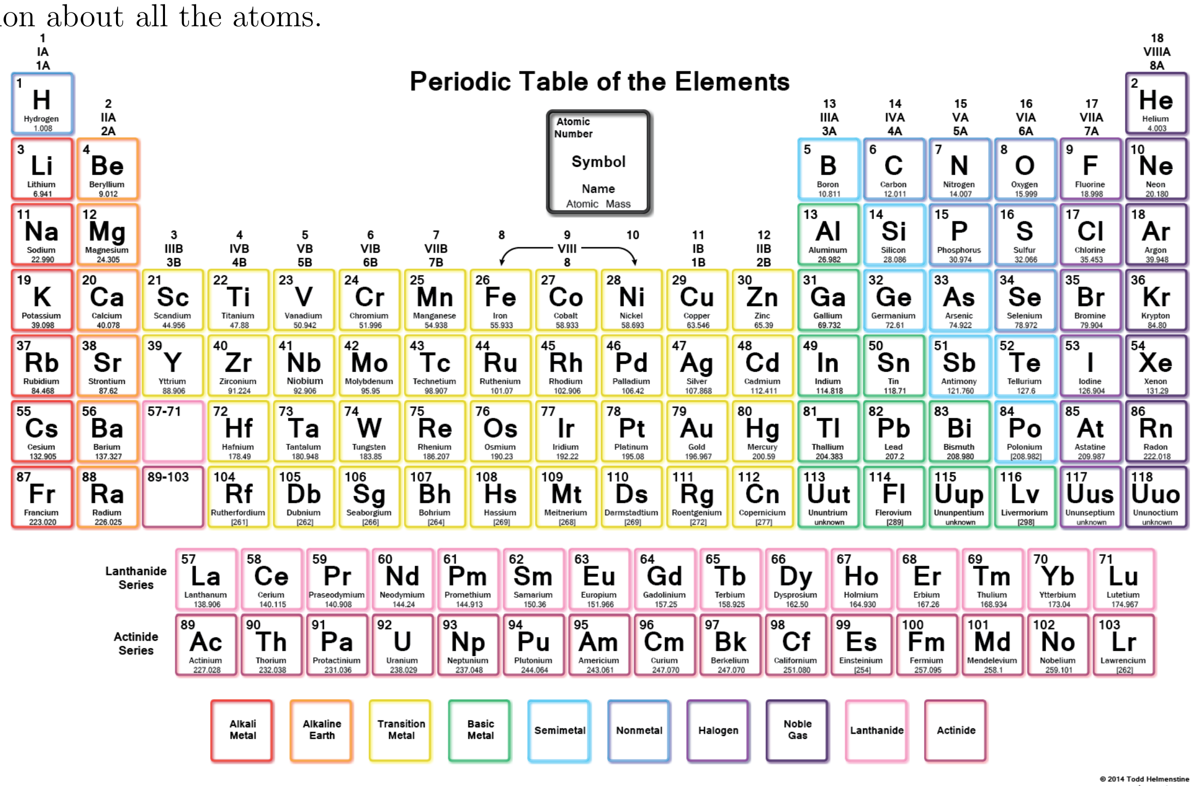
Molar mass. How do we know the amount of substance of some matter? It is really hard to resolve individual molecules, and there are too many of them (remember, the Avogadro number is $6 \cdot 10^{23}$) to count them one by one. But every molecule of a substance has the same mass, so $6.02 \cdot 10^{23}$ of these molecules have a particular mass which is characteristic of this substance. It is called molar mass and denoted by M. Mass is easy to measure and if we have m kilograms of substance with molar mass M, the amount of substance is

$$n = \frac{m}{M}.$$

Equation of state of ideal gas can now be written in terms of mass and molar mass:

$$pV = \frac{m}{M}RT$$

Molar mass from periodic table. How do we know the molar mass of some substance? As we have seen, molar mass is related to mass of molecules of the substance. Mass of a molecule is equal to sum of masses of atoms comprising this molecule. And masses of atoms can be found in the periodic table of elements, which contains a lot of useful information about all the atoms.



There is a simple algorithm of finding molar mass from periodic table. First of all, we need to locate atomic mass in the periodic table: it is the lowest number in each cell. For example, for the first element - hydrogen (H) we can see the atomic mass is 1.008 which could be rounded to 1. This is exactly the molar mass of a hydrogen atom, measured in gram/mole. So, if we take 1 mole of hydrogen atoms, or $6 \cdot 10^{23}$ hydrogen atoms, their mass will be $M(\mathrm{H}) \cdot 1$ mole $= 1$ g. If we take 1 mole of carbon atoms, their mass will be $M(\mathrm{C}) \cdot 1$ mole $= 12$ g (find carbon C in the table above and verify that its' atomic mass is about 12).

If instead of atoms we consider molecules, molar mass is sum of molar masses of atoms building the molecules. For example, a nitrogen molecule N_2 consists of two nitrogen atoms. Therefore, the molar mass of nitrogen molecules is twice the molar mass of nitrogen atoms:

$$M(\mathrm{N_2}) = 2 \cdot M(\mathrm{N}) = 2 \cdot 14 \text{ g/mole} = 28 \text{ g/mole}.$$

Let us do one more example. Consider a carbon dioxide molecule CO_2 which consists of a carbon atom and two oxygen atoms. From the periodic table we find that the molar mass of carbon atom C is 12 g/mole and that the molar mass of oxygen atom O is 16 g/mole. So we find molar mass of carbon dioxide:

$$M(\mathrm{CO_2}) = M(\mathrm{C}) + 2 \cdot M(\mathrm{O}) = 12 + 2 \cdot 16 \text{ g/mole} = 44 \text{ g/mole}.$$

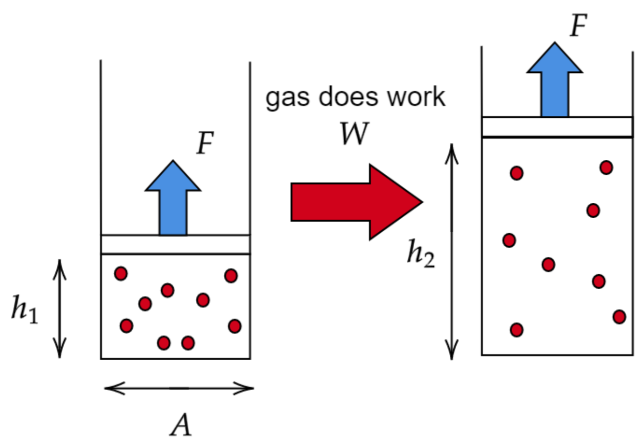
Work done by gas. Today we are considering a connection between mechanics and thermodynamics. We will learn how heat can be transferred to work using a gas.

First, a reminder: what is work? As you recall from mechanics, work is force times displacement. So in order to have work, we need to have force and displacement.

Consider our favorite setup: some amount of gas in a container under a piston. If cross section area of the container (and the piston) is A and gas pressure is p, the force with which gas acts on the piston follows from definition of pressure:

$$p = \frac{F}{A} \implies F = pA.$$

Let the gas be expanding, so the piston moves up by height $h = h_2 - h_1$ (see figure) above its initial position. Work done by the gas on the piston in this case is



$$W = Fh = pA(h_2 - h_1) = p(Ah_2 - Ah_1) = p(V_2 - V_1) = p\Delta V.$$

We used the fact that product of cross section area and height is volume of the gas (final volume is denoted V_2 and initial volume V_1) and denoted change of volume by ΔV. To sum up, we have derived the formula for work done by gas: work is done whenever volume is changed and for processes at constant pressure it is equal to pressure times change in volume:

$$W = p\Delta V.$$

Remember that work can be positive or negative: it is positive when force and displacement point in the same direction and negative when they point in the opposite directions. Let us consider two possible cases to show that our formula for work of the gas does get this right. When volume changes, it either increases or decreases. When volume increases, force on the piston and displacement of the piston both point up in our figure above. Therefore work should be positive - and from our formula $W = p\Delta V > 0$ because $\Delta V > 0$ (and p is always positive). On the other hand when volume decreases, displacement is in the opposite direction (down on our figure) than the force (which still points up). Therefore work should be negative and from our formula $W = p\Delta V < 0$ because $\Delta V < 0$.

Homework

1. There is a 1 liter bottle filled with water at 27°C. The water is liquid at this temperature because there is attracting force between the molecules. Imagine that we have suddenly "turned off" this attracting force. What is the pressure in the bottle now? *Hint: mass of 1 liter of water is 1 kg, molar mass of water is 18 grams/mole.*

2. Find molar mass of molecular oxygen O_2 using periodic table. Using it, find the mass of oxygen in a 10 liter cylinder if it has temperature T=13°C and pressure $P = 9 \cdot 10^6$ Pa (note that it is 90x the normal atmospheric pressure!). For how long can the oxygen in this cylinder sustain a scuba diver, if an average person needs to inhale about 2 grams of oxygen per minute?

3. There is a cylinder with a piston. The mass of the piston is 100 kg, its area is 10 cm^2. The cylinder contains 32 grams of oxygen at $T_1 = 273$ K. The cylinder is heated up to $T_2 = 373$ K. How does the piston position change? How does the potential energy of the piston change? What work is done by the gas? Neglect atmospheric pressure. *Hint: what is the gas pressure? Does it change?*

***4.** How much hydrogen H_2 (in grams) is in a cylinder with a piston if it performs work of 400 J being heated from 250 K to 680 K? The gas pressure was maintained constant.