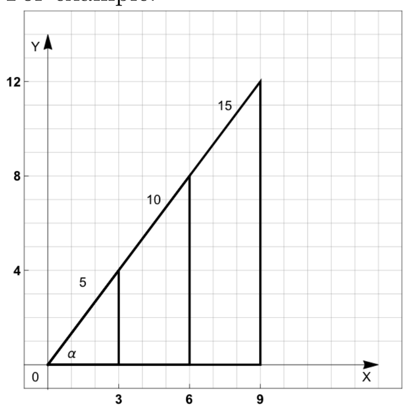
MATH 7: HANDOUT 21
TRIGONOMETRY 2: LAW OF SINES

Tangent $\tan(\alpha)$

Now we can also define the 3rd trigonometric ratio:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypotenuse}}{\text{adjacent side/hypotenuse}} = \frac{\text{opposite side}}{\text{adjacent side}}$$

For example:



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$
$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$
$$\tan\alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{8}$$

Trigonometric Functions							
Function	Notation	Definition	0°	30°	45°	60°	90°
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Trigonometric Identities and Laws of Sines

The most prominent trigonometric identity is:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

Let us try to derive it:

- A right triangle with hypotenuse c and an angle α is given. Express the remaining 2 sides (a and b) of triangle using only c and α.
- Using expressions obtained for a and b, express the hypotenuse c and simplify.

Law of Sines: Given a triangle ΔABC with sides a, b, and c, the following is always true:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Proof: To see why the Law of Sines is true, refer to the Figure 1. The height of the triangle $h = b\sin C$, and therefore the area of the triangle is $S = \frac{1}{2}ab\sin C$. Similarly, $S = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$. Thus,

$$bc\sin A = ac\sin B = ab\sin C$$

Dividing by abc, we get:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

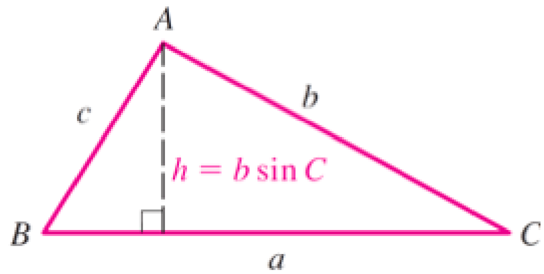


Figure 1. Law of Sines

Homework

1. If a right triangle ΔABC has sides $AB = 3 * \sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.
2. The area of a right triangle is 36 square meters. The legs of the triangle have the ratio of $2:9$. Find the hypotenuse of the triangle.
3. In a triangle ΔABC, we have $\angle A = 40°, \angle B = 60°$, and $AB = 2$ cm. What is the remaining angle and side lengths? (Hint: Use Law of sines)
4. In an isosceles triangle, the angle between the equal sides is equal to $30°$, and the height is 8. Find the sides of the triangle.
5. A right triangle ΔABC is positioned such that A is at the origin, B is in the 1st quadrant ($B_x > 0$ and $B_y > 0$) and C is on the positive horizontal axis ($C_x > 0$ and $C_y = 0$). If length of side AB is 1, and AB makes a $35°$ angle with positive x axis, what are the coordinates of B?
6. Consider a parallelogram $ABCD$ with $AB = 10$, $AD = 4$ and $\angle BAD = 50°$. Find the length of diagonal AC.
7. A regular heptagon (7 sides) is inscribed into a circle of radius 1.
   (a) What is the perimeter of the heptagon?
   (b) What is the area of the heptagon?
8. In the trapezoid below, $AD = 5$ cm, $AB = 2$ cm, and $\angle A = \angle D = 70°$. Find the length BC and the diagonals. [You can use: $\sin(70°) \approx 0.94, \cos(70°) \approx 0.34$.]
9. To determine the distance to the enemy gun (point E in the figure below), the army unit placed two observers (points A, B in the figure below) and asked each of them to measure the angles using a special instrument. The results of the measurements are shown below. If it is known that the distance between the observers is 400 meters, can you determine how far away from observer A is the enemy gun?

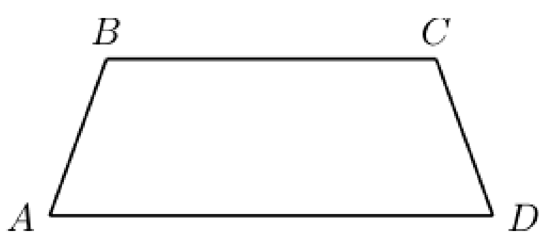


Figure 2. Problem 8

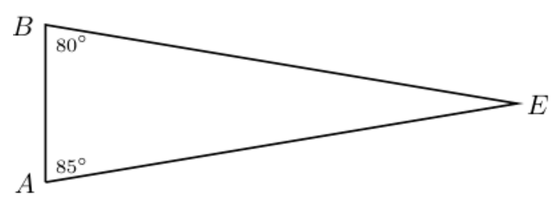


Figure 3. Problem 9