## MATH 5 <br> GEOMETRY 5.

## Similar figures

Two figures are called similar if one of them can be obtained from the other by rescaling, or stretching (equally in all directions).


In similar figures all angles are the same, and all lenghts are increased in the same proportion. For example, if triangles $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$ are similar, then

$$
\begin{aligned}
& \angle A=\angle A^{\prime}, \quad \angle B=\angle B^{\prime} \quad \angle C=\angle C^{\prime} \\
& A^{\prime} B^{\prime}=k A B, \quad A^{\prime} C^{\prime}=k A C, \quad B^{\prime} C^{\prime}=k B C
\end{aligned}
$$

The number $k$ is called the similarity coefficient.

## Similarity test for triangles

AAA Rule. If three angles of one triangle are equal to corresponding angles of another triangle, then the triangles are similar.

For example, if $A B C, A^{\prime} B^{\prime} C^{\prime}$ is a right triangles with $\angle C=\angle C^{\prime}=90^{\circ}, \angle A=\angle A^{\prime}=30^{\circ}$, then these triangles are similar.

Another example is shown in the figure to the right. Let lines $l=A B$, $l^{\prime}=A^{\prime} B^{\prime}$ be parallel. Then triangles $A B C$ and $A^{\prime} B^{\prime} C$ are similar: indeed, by the theorem about alternate angles, we have $\angle A=\angle A^{\prime}$, $\angle B=\angle B^{\prime}$.


## Homework

1. Find the missing lengths in the second triangle (triangles are similar).

2. Find the lengths $x, y$ in the figure (lines $l, m$ are parallel).

3. The figure to the right shows some of the angles and sides in a trapezoid. The height $h$ of this trapezoid is equal to $\sqrt{3}$. Find all other angles, sides and area of the trapezoid. (Hint: you will need Pythagorean theorem!)

4. The figure shows right triangle $A B C: \angle C=90^{\circ}$, with $A C=4 \mathrm{~cm}, A B=5 \mathrm{~cm}$. The line $C D$ is the altitude, i.e., it is perpendicular to side $A B$.
(a) Show that triangles $A B C$ and $A C D$ are similar.
(b) Find lengths $B C, C D$.

5. Given a rectangle with sides 2 cm and 4 cm , can you:
(a) Cut it into three pieces that can be rearranged to get a right triangle?
(b) Cut it into three pieces that can be rearranged to get a square?
6. In a rhombus, two diagonals are 16 cm and 12 cm . Find the area and perimeter of the rhombus.
7. Solve the equation $7 x-11=5 x-3$.
